5.1 Mathematical Models and Calculations

The aerodynamic design of wind turbine rotors requires more than knowledge of the elementary physical laws of energy conversion. The designer faces the problem of finding the relationship between the actual shape of the rotor, e.g. the number of rotor blades or the airfoil of its blades, and its aerodynamic properties.

As with most technical designs, this design process is carried out iteratively, in practice. In the beginning, there is the concept of a rotor which promises to have certain desired properties. A calculation is then carried out for this configuration and checked to see the extent to which the expected result is actually obtained. As a rule, the results will not be completely satisfactory in the first instance but the mathematical/physical model provides an insight into how the given parameters of the rotor design have affected the end result. This provides an opportunity for improving the design by applying the appropriate corrections.

It would exceed the scope of this book to describe the mathematical models currently used in designing the aerodynamics of wind turbine rotors. Nevertheless, the main approaches to the theory of rotor aerodynamics will be explained since they are useful in understanding the results of the calculations, and thus the shape of wind turbine rotors.

The fundamental theoretical approaches are based on the work of numerous aerodynamicists who, in the 1920s, were faced with the task of providing reliable and scientifically founded calculation tools for aircraft engineers who had been working on a rather more empirical basis up to that point. It was the task of finding the aerodynamically optimum wing, in particular, which was of crucial significance to the advancement of aviation and which, ultimately, resulted in the evolution of a special discipline of applied flow mechanics, the so-called "wing theory". Notable names in this field are Prandtl, Glauert, Multhopp, Schlichting and Truckenbrodt. Together with this wing theory, the theoretical models of propeller and turbine design calculation form the starting point for the calculation of the aerodynamics of wind turbine rotors.

It is German aerodynamicist Albert Betz's merit to have formulated not only the basic physical laws of energy conversion but also a complete theory of the wind rotor. In the years to follow, this theory was developed further by numerous other authors. Among others, it was Ulrich Hütter who distinguished himself by significantly advancing and refining Betz's theory in the years between 1940 and 1942 [1]. These efforts have been furthered in recent decades by intensive work conducted on helicopter rotors. The Americans Wilson and Lissaman have published computation methods designed especially for use on computers [2].

Betz's simple momentum theory is based on the modelling of a two-dimensional flow through the actuator disc (s. Chapter 4). The airflow is slowed down and the flow lines are deflected only in one plane (Fig. 5.1).

In reality, however, a rotating converter, a rotor, will additionally impart a rotating motion, a *spin*, to the rotor *wake*. To maintain the angular momentum, the spin in the wake must be opposite to the torque of the rotor.

The energy contained in this spin reduces the useful proportion of the total energy content of the air stream at the cost of the extractable mechanical energy so that, in the extended momentum theory, taking into consideration the rotating wake, the power coef-

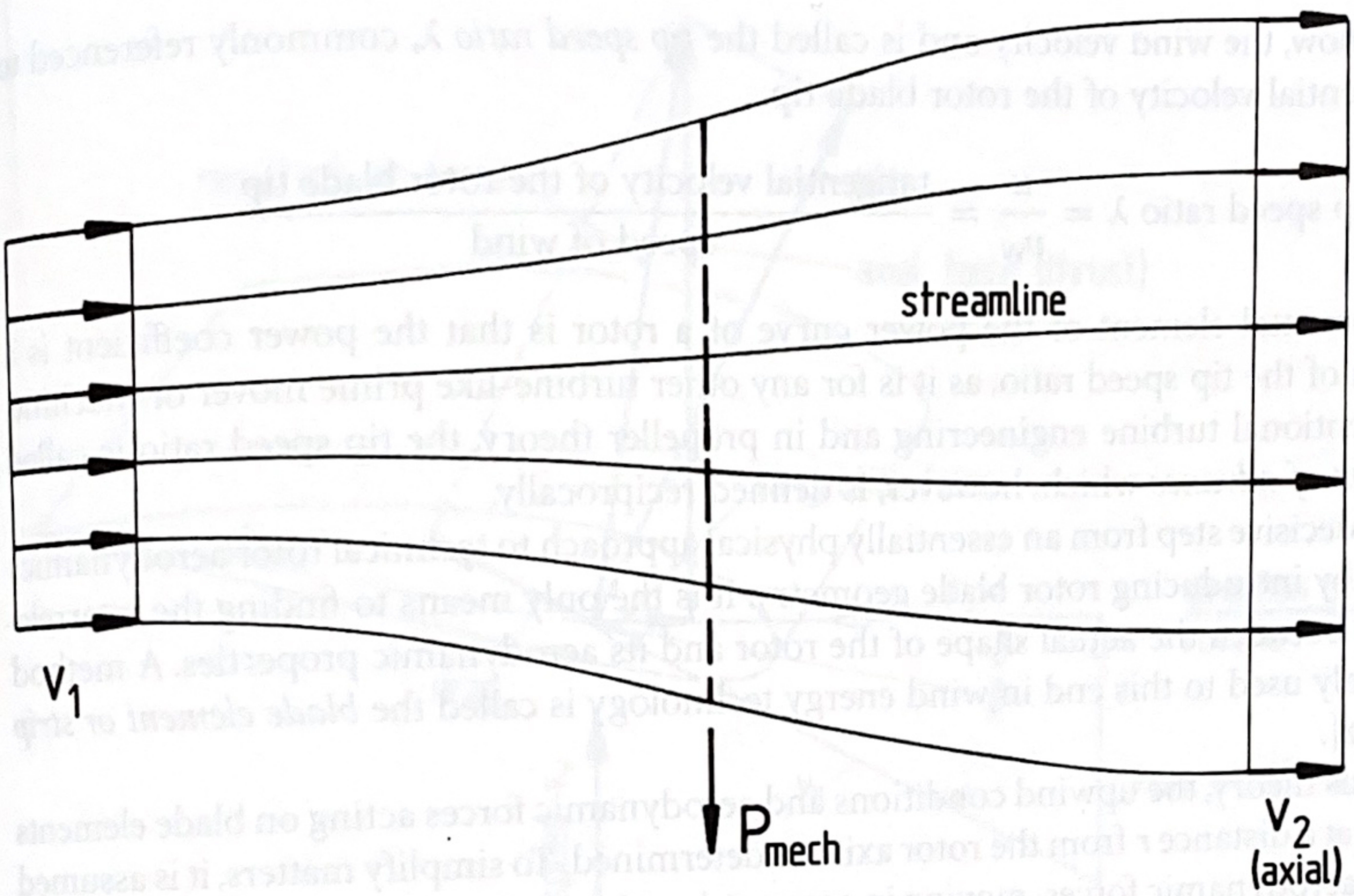

Figure 5.1. Flow model of Betz's momentum theory

ficient of the turbine must be smaller than the value according to Betz (Fig. 5.2). Moreover, the power coefficient now becomes dependent on the ratio between the energy components from the rotating motion and the translatorial motion of the air stream. This ratio is determined by the tangential velocity of the rotor blades in relation to the undisturbed

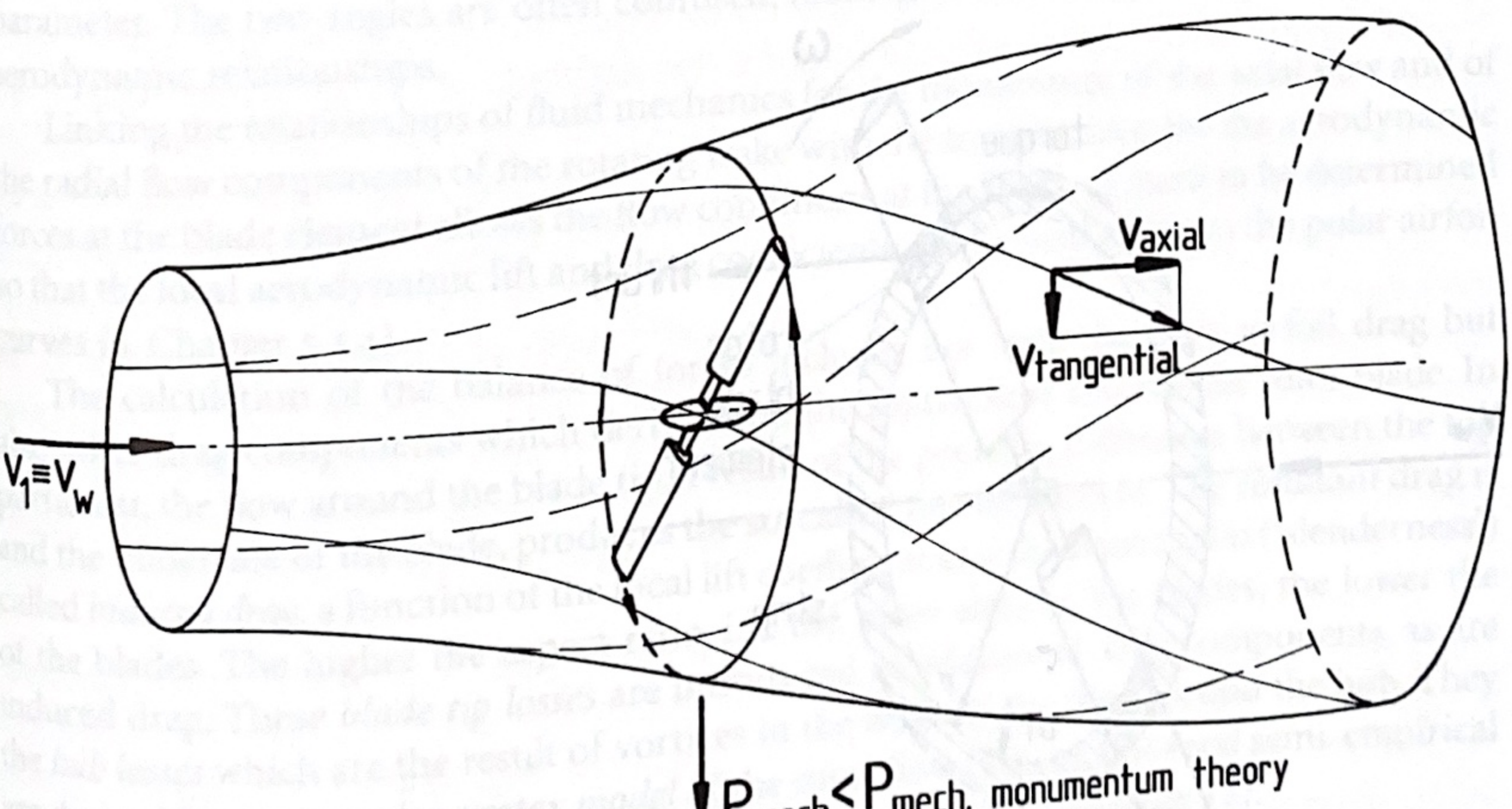

Figure 5.2. Extended momentum theory, taking into consideration the rotating rotor wake

axial airflow, the wind velocity and is called the *tip speed ratio* λ, commonly referenced to the tangential velocity of the rotor blade tip.

$$\text{Tip speed ratio } \lambda = \frac{u}{v_W} = \frac{\text{tangential velocity of the rotor blade tip}}{\text{speed of wind}}$$

A fundamental element of the power curve of a rotor is that the power coefficient is a function of the tip speed ratio, as it is for any other turbine-like prime mover or machine. In conventional turbine engineering and in propeller theory, the tip speed ratio is called *coefficient of advance* which, however, is defined reciprocally.

The decisive step from an essentially physical approach to technical rotor aerodynamics is taken by introducing rotor blade geometry. It is the only means to finding the interrelationship between the actual shape of the rotor and its aerodynamic properties. A method commonly used to this end in wind energy technology is called the *blade element or strip theory* [2].

In this theory, the upwind conditions and aerodynamic forces acting on blade elements rotating at a distance r from the rotor axis are determined. To simplify matters, it is assumed that the aerodynamic forces, moving in concentric strips, do not interfere with one another (Fig. 5.3). The blade element is formed by the local rotor blade chord (aerodynamic airfoil) and the radial extent of the element dr.

The airfoil cross-section at radius r is set at a local blade pitch angle ϑ with respect to the rotor plane of rotation (Fig. 5.4). The axial free stream velocity v_a in the rotor plane and the tangential speed u at the radius of the blade cross-section combine to form a resultant flow velocity v_r. Together with the airfoil chord line, it forms the local aerodynamic *angle of attack* α. For the benefit of those readers unfamiliar with aerodynamics, the difference

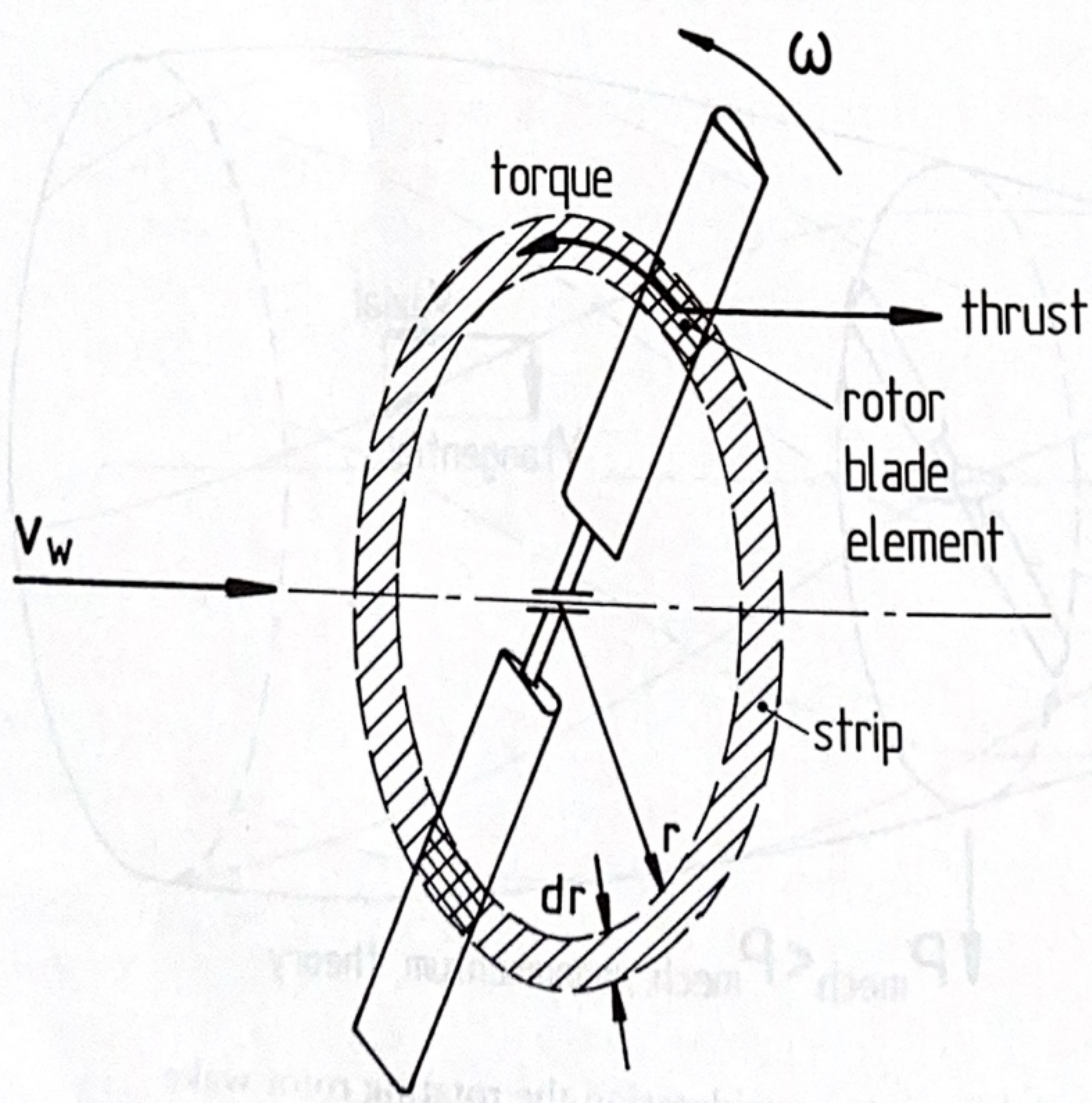

Figure 5.3. Strip theory model

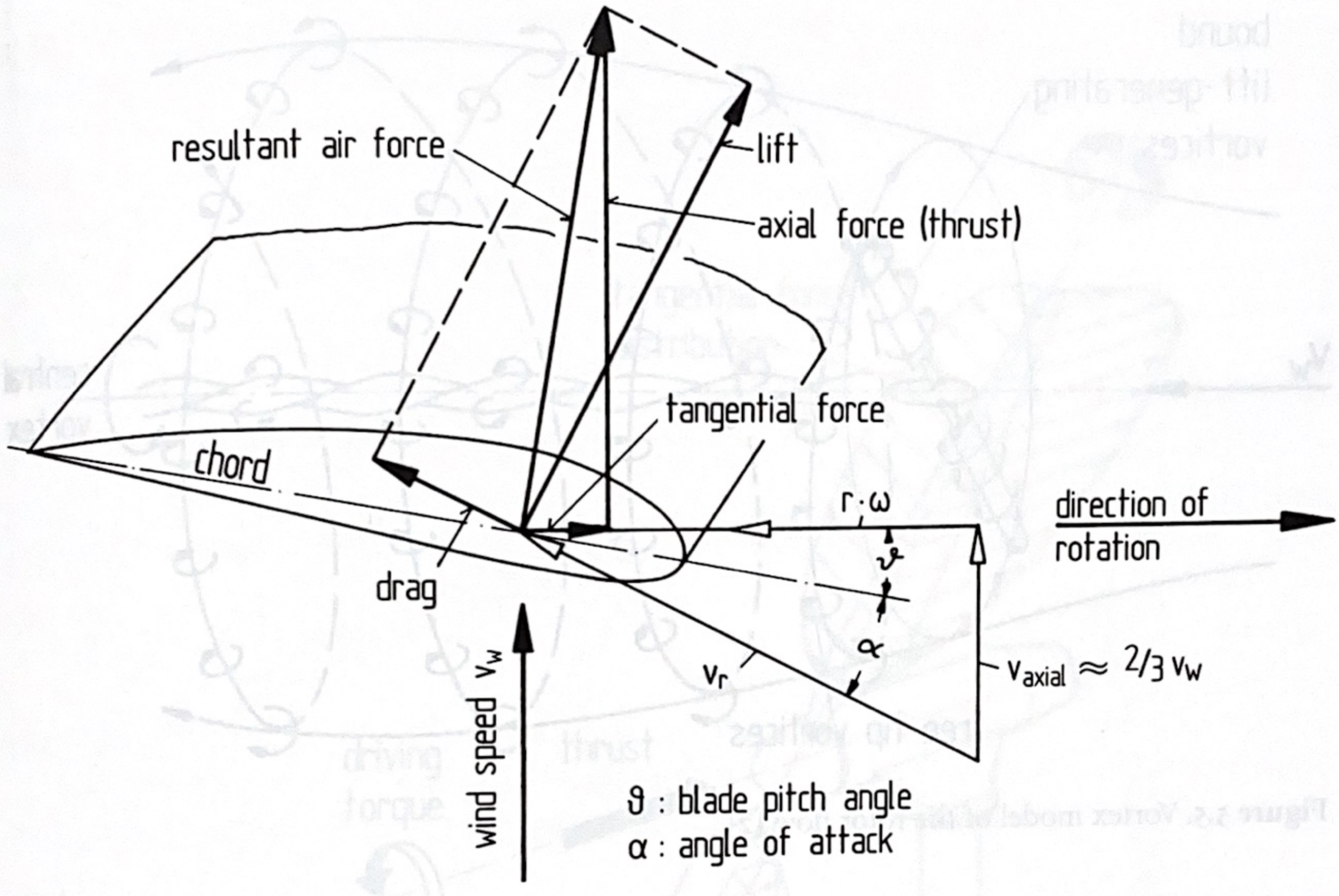

Figure 5.4. Flow velocities and aerodynamic forces at the airfoil cross-section of a blade element

between the aerodynamic angle of attack α and the blade pitch angle ϑ should be noted: the angle of attack is an aerodynamic parameter and the blade pitch angle is a design parameter. The two angles are often confused, making it more difficult to understand the aerodynamic relationships.

Linking the relationships of fluid mechanics for the momentum of the axial flow and of the radial flow components of the rotating wake with the formulations for the aerodynamic forces at the blade element allows the flow conditions at the blade element to be determined so that the local aerodynamic lift and drag coefficients can be read off from the polar airfoil curves (s. Chapter 5.3.4).

The calculation of the balance of forces includes not only the pure airfoil drag but also other drag components which derive from the spatial flow around the rotor blade. In particular, the flow around the blade tip, a result of the pressure difference between the top and the underside of the blade, produces the so-called *free tip vortices*. The resultant drag is called *induced drag*, a function of the local lift coefficient and the *aspect ratio* ('slenderness') of the blades. The higher the aspect ratio, i. e. the more slender the blades, the lower the induced drag. These *blade tip losses* are introduced as additional drag components, as are the *hub losses* which are the result of vortices in the wake of the flow around the hub. They are derived from a complex *vortex model* of the rotor flow (Fig. 5.5). Several semi-empirical approaches for these vortex losses have been described in the literature [2].

With its calculation of the local aerodynamic lift and drag coefficients, the blade element theory provides the distribution of aerodynamic forces over the length of the blade. This is usually divided into two components: one in the plane of rotation of the rotor —

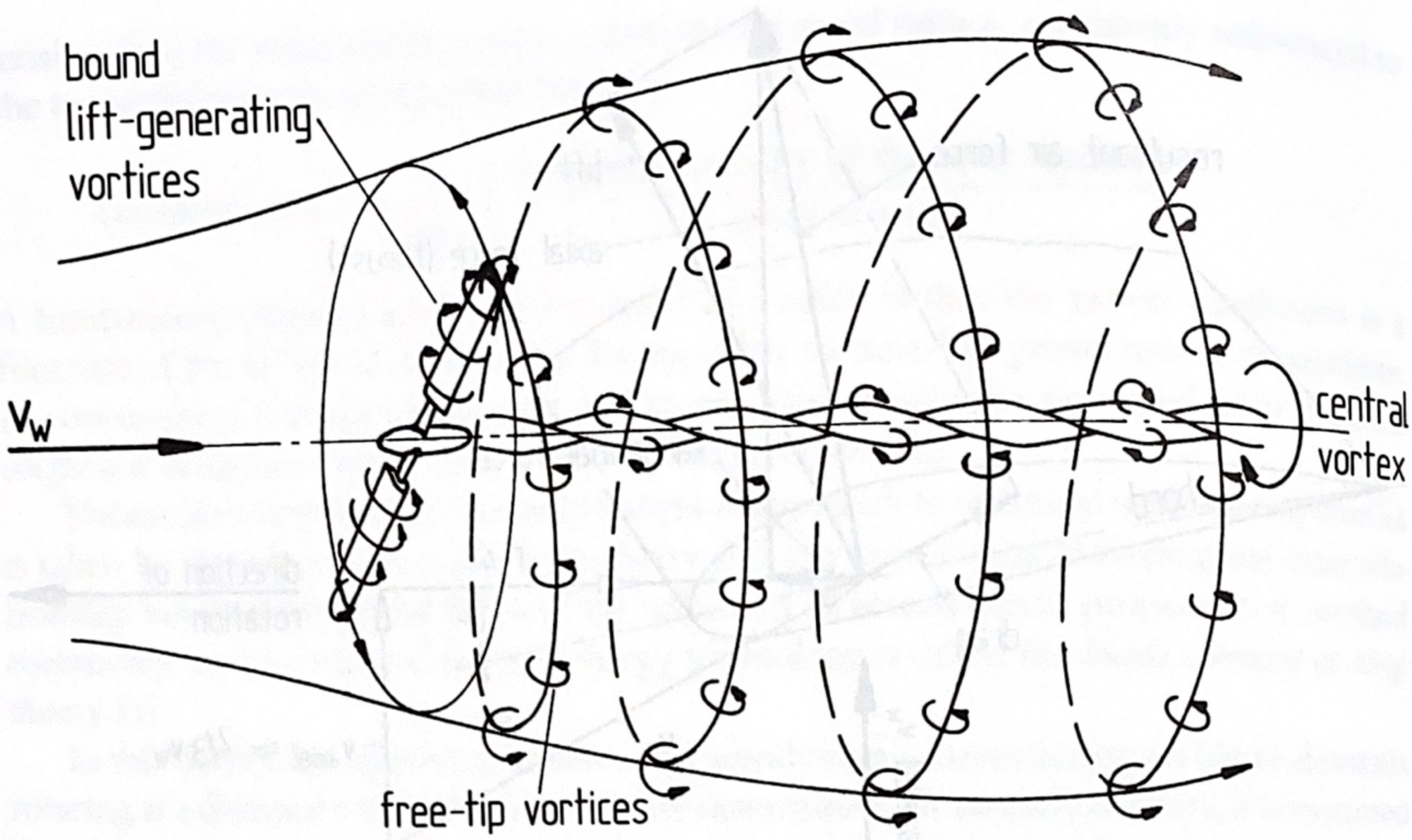

Figure 5.5. Vortex model of the rotor flow [2]

the tangential force distribution, and one at right angles to it — the thrust distribution (Fig. 5.6). Integrating the tangential force distribution over the rotor radius provides the driving torque of the rotor and, with the rotational speed of the rotor, the rotor power or power coefficient, respectively. Integrating the thrust distribution yields the total rotor thrust for instance to the tower. The blade element or strip theory thus provides both the rotor power and the steady-state aerodynamic loading for a given blade geometry.

Taking the rotor power characteristic, i.e. the variation of the power coefficient as a function of the tip speed ratio, as an example, the approximation of the theoretical models to reality can be illustrated retrospectively (Fig. 5.7). Referred to the power rating of the air stream, the simple momentum theory by Betz provides the ideal constant power coefficient of 0.593 which is independent of the tip speed ratio. Taking into consideration the angular momentum in the rotor wake shows that the power coefficient becomes a function of the tip speed ratio. It is only when the tip speed ratios become infinitely high that the power coefficient approaches Betz's ideal value. Introducing the aerodynamic forces acting on the rotor blades, and particularly the aerodynamic drag, further reduces the power coefficient; in addition, the power coefficient now exhibits an optimum value at a certain tip speed ratio.

The aerodynamic rotor theory based on the momentum theory and on the blade element theory, yields the real rotor power curve with good approximation. Nevertheless it should be kept in mind that the momentum theory as well as the blade element model include several simplifications which limit their validity to a disc shaped wind energy converter. Sometimes the momentum theory is therefore called "disc actuator theory". The propeller type rotor is very close to this model, but not all the other unconventional designs acting as wind energy converters, are disc-shaped devices, converting the wind energy to mechanical energy in one step.

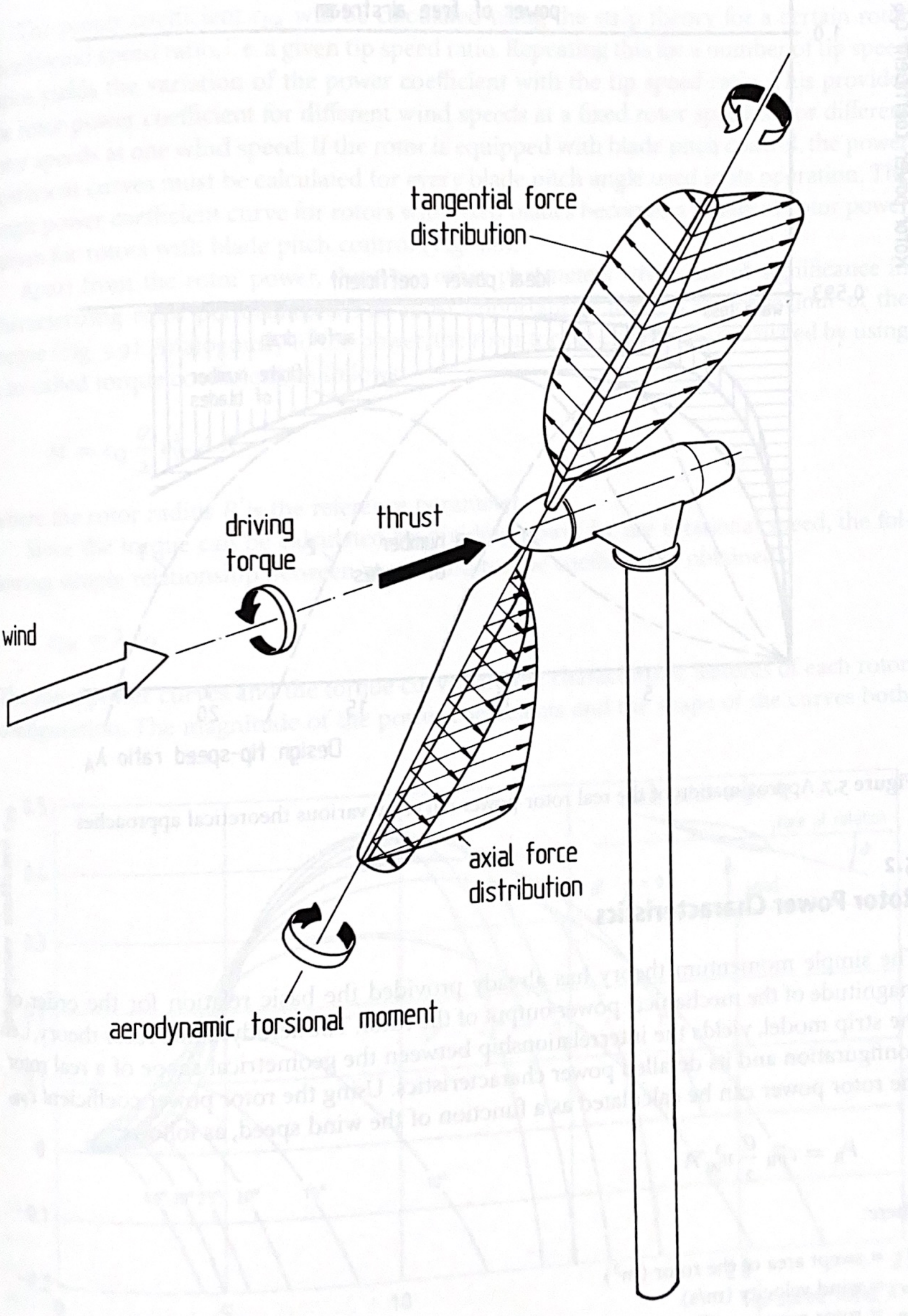

Figure 5.6. Distribution of aerodynamic forces over the blade length and total rotor forces and torques

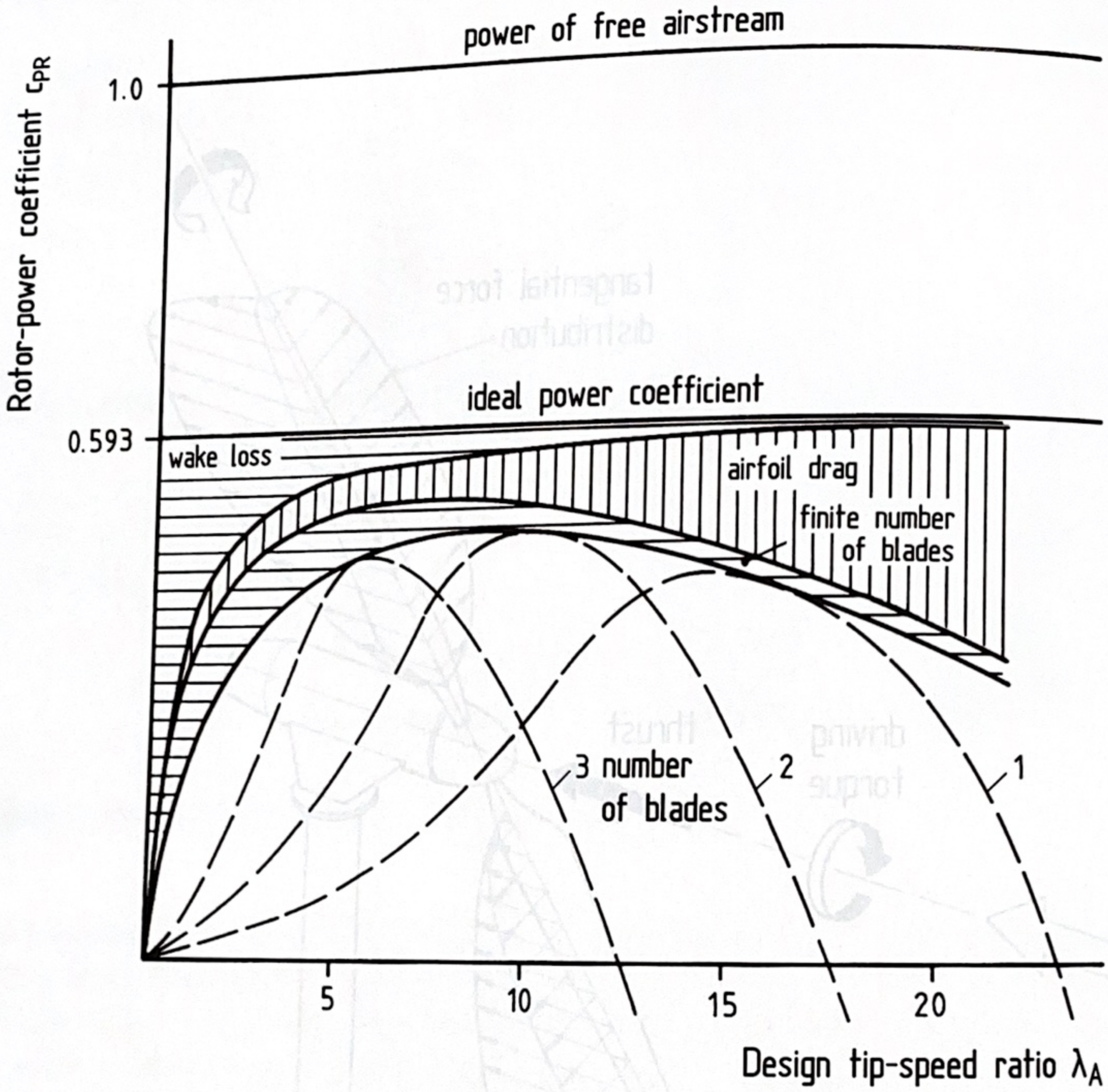

Figure 5.7. Approximation of the real rotor power curve by various theoretical approaches

5.2 Rotor Power Characteristics

The simple momentum theory has already provided the basic relation for the order of magnitude of the mechanical power output of the rotor. The aerodynamic rotor theory, i.e. the strip model, yields the interrelationship between the geometrical shape of a real rotor configuration and its detailed power characteristics. Using the rotor power coefficient c_{PR}, the rotor power can be calculated as a function of the wind speed, as follows:

$$P_R = c_{PR} \frac{\varrho}{2} v_W^3 A$$

where:

A = swept area of the rotor (m^2)
v_W = wind velocity (m/s)
c_{PR} = rotor power coefficient (—)
ϱ = air density (kg/m^3 at MSL)
P_R = rotor power (W)

The power coefficient c_{PR} will be calculated using the strip theory for a certain rotor speed/wind speed ratio, i. e. a given tip speed ratio. Repeating this for a number of tip speed ratios yields the variation of the power coefficient with the tip speed ratio. This provides the rotor power coefficient for different wind speeds at a fixed rotor speed or for different rotor speeds at one wind speed. If the rotor is equipped with blade pitch control, the power coefficient curves must be calculated for every blade pitch angle used in its operation. The single power coefficient curve for rotors with fixed blades becomes a family of rotor power curves for rotors with blade pitch control (Fig. 5.8).

Apart from the rotor power, there are other parameters which are of significance in characterizing rotor performance. The most important of these is the behaviour of the torque (Fig. 5.9). Analogously to the power, the rotor torque can also be calculated by using a so-called torque coefficient, as follows:

$$M = c_Q \frac{\varrho}{2} v_W^2 A R$$

where the rotor radius R is the reference parameter.

Since the torque can be calculated by dividing power by the rotational speed, the following simple relationship between power and torque coefficient is obtained:

$$c_{PR} = \lambda c_Q$$

The rotor power curves and the torque curves are the characteristic features of each rotor configuration. The magnitude of the power coefficients and the shape of the curves both

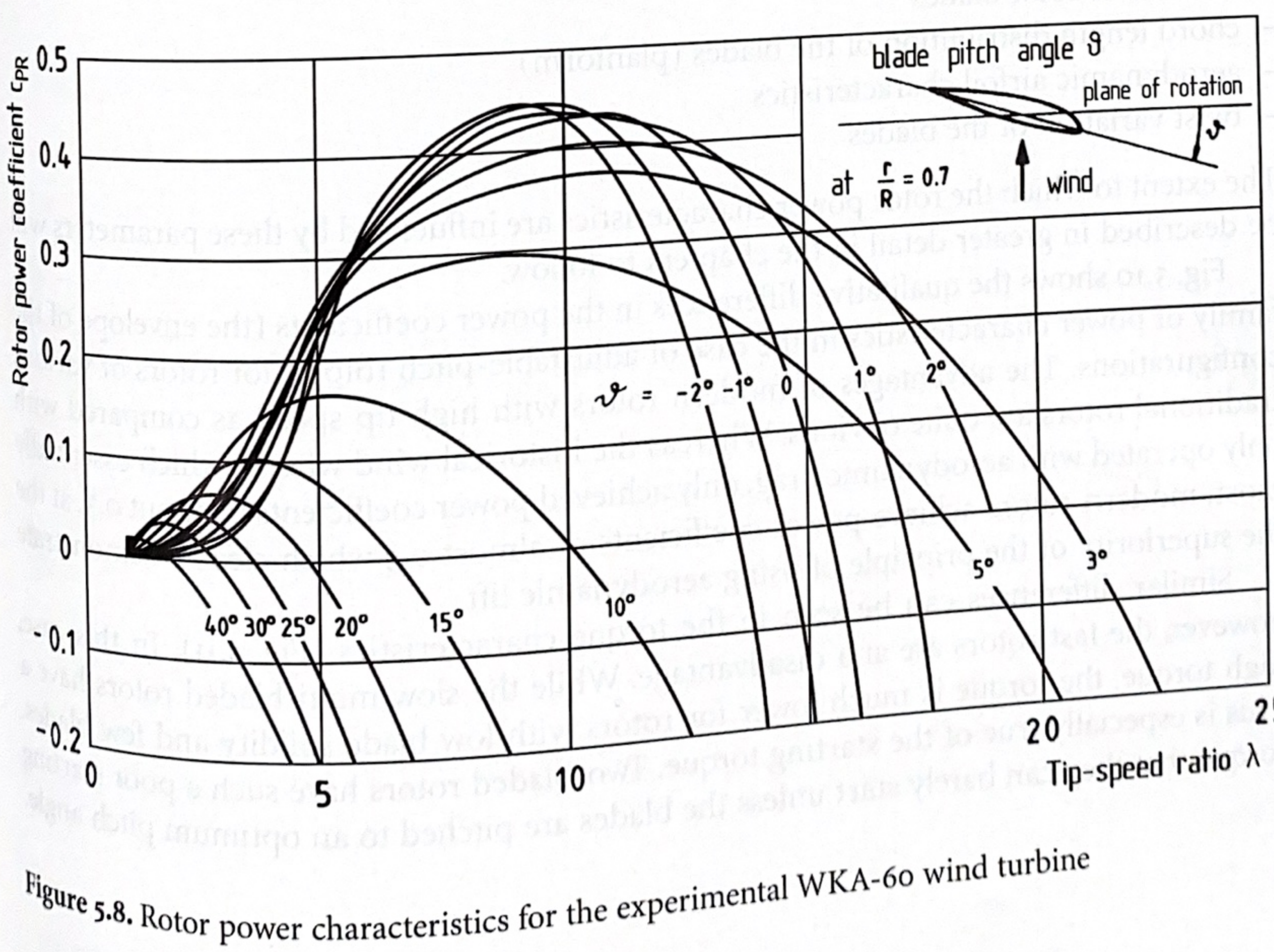

Figure 5.8. Rotor power characteristics for the experimental WKA-60 wind turbine

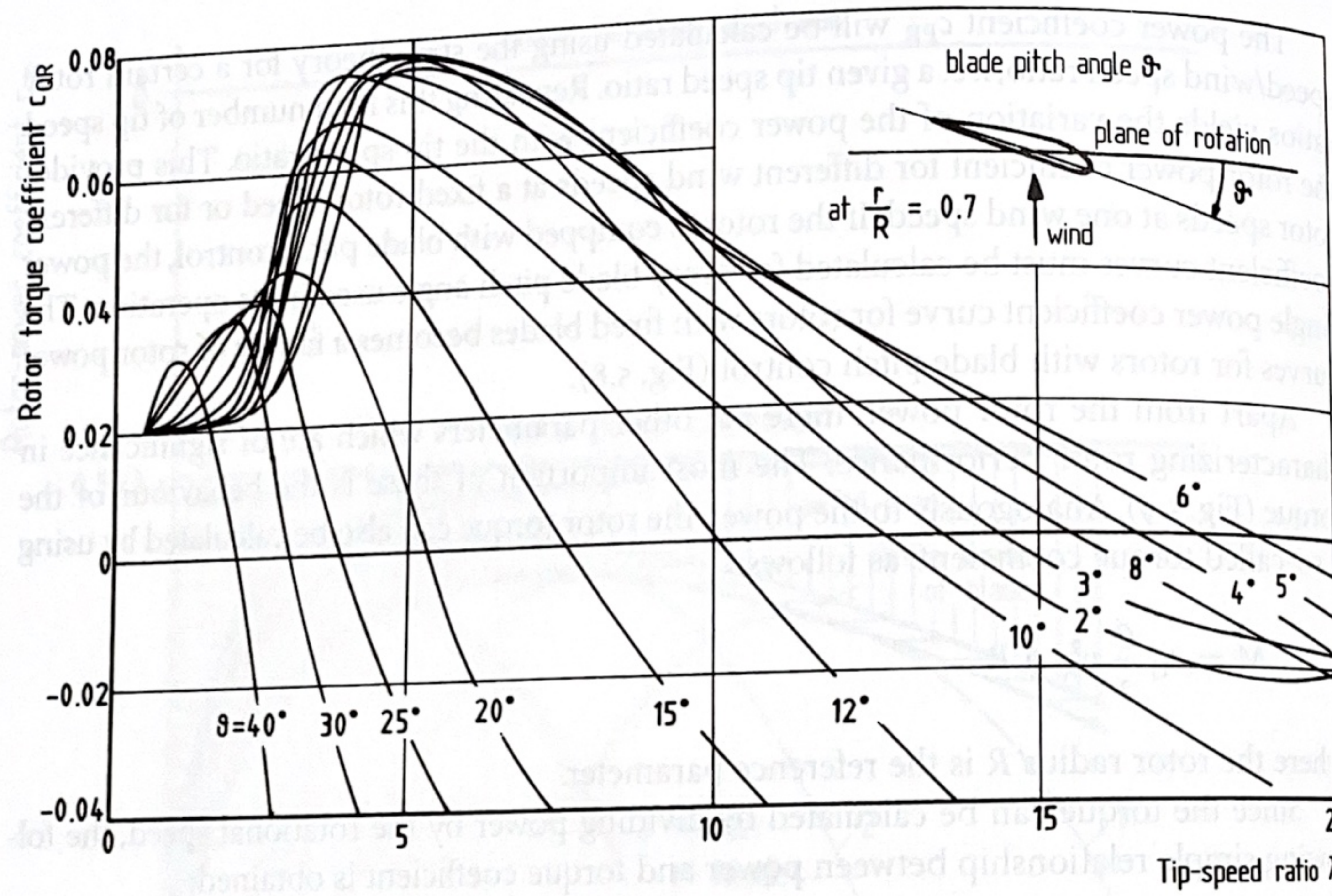

Figure 5.9. Rotor torque characteristics for the WKA-60

show distinct differences. The main parameters dominating the c_{PR} map are:

- number of rotor blades
- chord length distribution of the blades (planform)
- aerodynamic airfoil characteristics
- twist variation of the blades

The extent to which the rotor power characteristics are influenced by these parameters will be described in greater detail in the chapters to follow.

Fig. 5.10 shows the qualitative differences in the power coefficients (the envelope of the family of power characteristics in the case of adjustable-pitch rotors) for rotors of various configurations. The advantages of modern rotors with high tip speed as compared with traditional rotors are quite obvious. Whereas the historical wind wheels, which essentially only operated with aerodynamic drag, only achieved power coefficients of about 0.3, at the most, modern rotors achieve power coefficients of almost 0.5 which clearly demonstrate the superiority of the principle of using aerodynamic lift.

Similar differences can be seen in the torque characteristics (Fig. 5.11). In this case, however, the fast rotors are at a disadvantage. While the slow multi-bladed rotors have a high torque, the torque is much lower for rotors with low blade solidity and few blades. This is especially true of the starting torque. Two-bladed rotors have such a poor starting torque that they can barely start unless the blades are pitched to an optimum pitch angle.

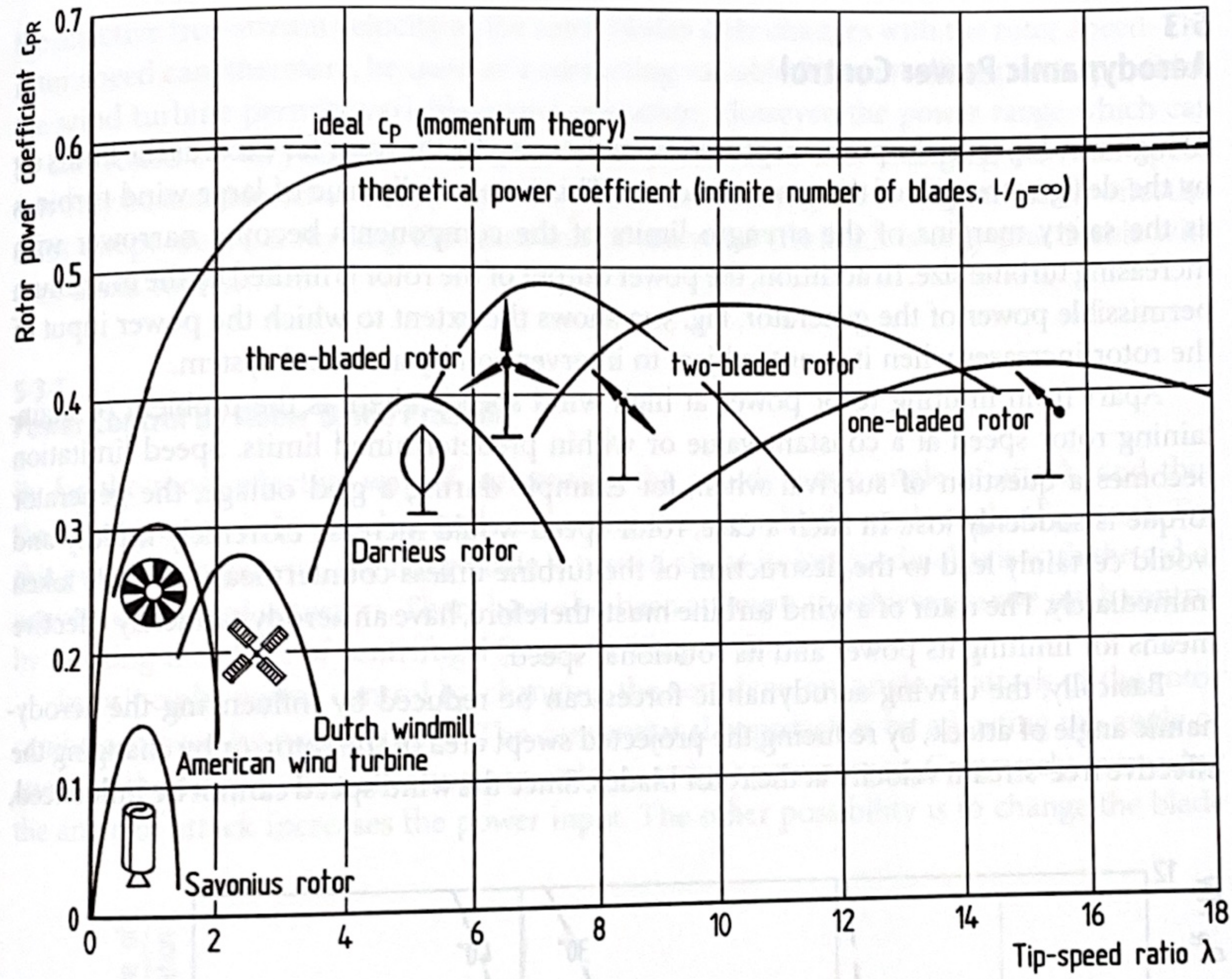

Figure 5.10. Power coefficients of wind rotors of different designs [2]

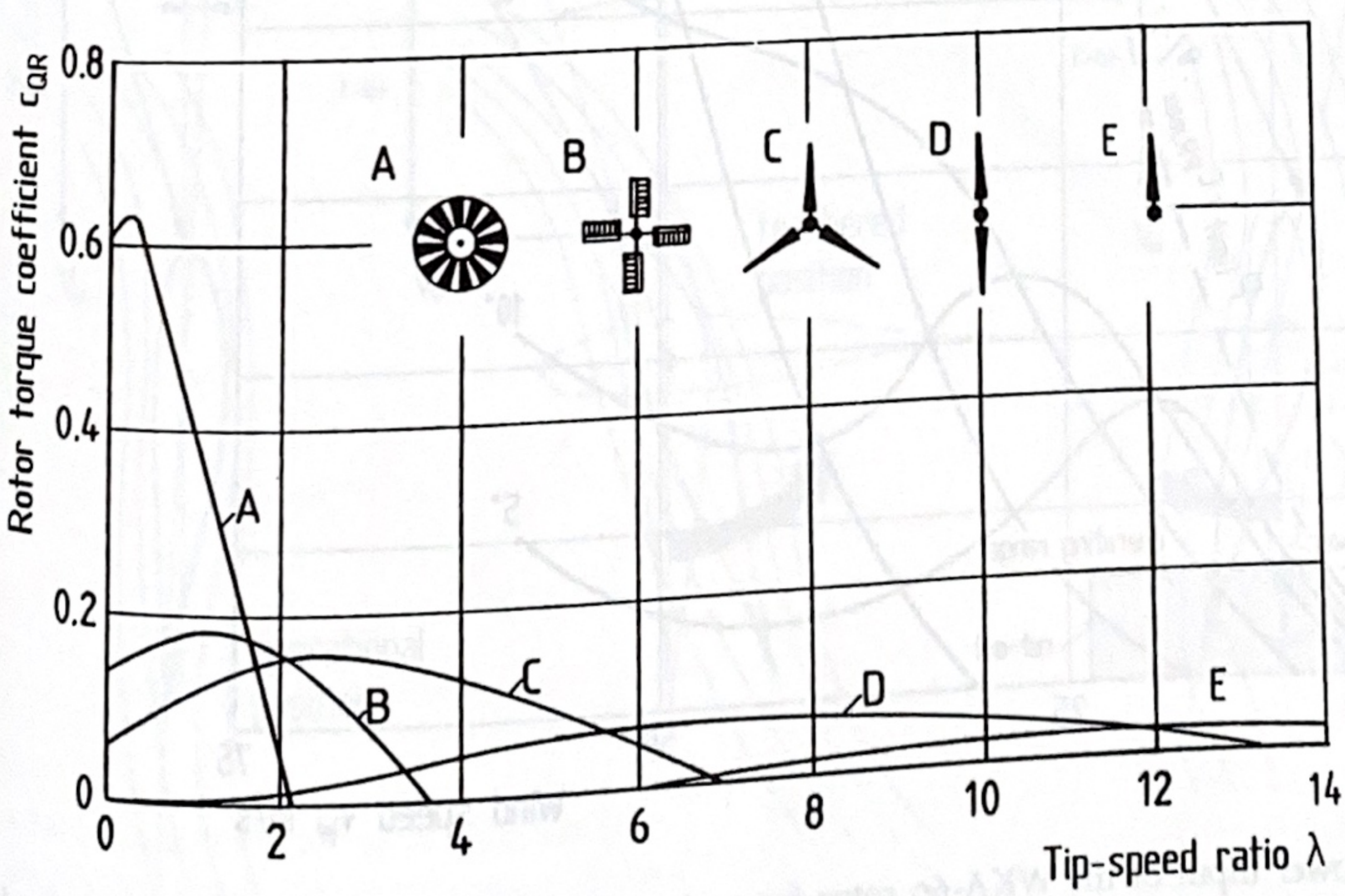

Figure 5.11. Torque coefficients of wind rotors of different designs [2]